

On the strong coupling scale in Higgs G-inflation

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Higgs G-inflation is an inflation model that takes advantage of a Galileon-like derivative coupling. It is a non-renormalizable operator and is strongly coupled at high energy scales. Perturbative analysis does not have a predictive power any longer there. In general, when the Lagrangian is expanded around the vacuum, the strong coupling scale is identified as the mass scale that appears in non-renormalizable operators. In inflationary models, however, the identification of the strong coupling scale is subtle, since the structures of the kinetic term as well as the interaction itself can be modified by the background inflationary dynamics. Therefore, the strong coupling scale depends on the background. In this letter, we evaluate the strong coupling scale of the fluctuations around the background in the Higgs G-inflation including the Nambu-Goldstone modes associated with the symmetry breaking. We find that the system is sufficiently weakly coupled when the scales which we now observe exit the horizon during inflation and the observational predictions with the semiclassical treatment are valid. However, we also find that the inflaton field value at which the strong coupling scale and the Hubble scale meet is less than the Planck scale. Therefore, we cannot describe the model from the Planck scale, or the chaotic initial condition.

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I. INTRODUCTION

The identification of inflaton is one of the remaining key pieces in the inflationary cosmology. Since the Higgs field is its unique candidate in the standard model (SM), investigating the possibilities of inflation driven by the SM Higgs field is one of the important issues for both high energy physics and cosmology. Among many proposals of the Higgs inflation models [1–3], Higgs G-inflation [4], where the kinetic term of the Higgs field is dominated by a Galileon-like term [5–8] during inflation, has a distinct characteristic. It can generate a large gravitational wave background,¹ violating the (standard) Lyth bound [10, 11]. Moreover, it breaks the standard consistency relation between the tensor-to-scalar ratio and the spectral tilt of the power spectrum of the tensor modes, $n_T = -r/8$, without introducing any non-trivial direct couplings between inflaton and gravity.

The Galileon-like term that we introduce here² $\phi \square \phi (\partial \phi)^2 / 2M^4$ contains a mass parameter $M \sim 10^{13}$ GeV [4], which is the strong coupling scale or the ultraviolet (UV) cutoff of the theory around the vacuum. Inflation takes place at $\phi \gtrsim \sqrt{MM_{\text{Pl}}}$ with the Hubble parameter $H \gtrsim M$. Here $M_{\text{Pl}} (> M)$ is the reduced Planck mass and ϕ is the inflaton or the physical Higgs field in the unitary gauge. As is addressed in the case of the Higgs inflation with non-minimal coupling to gravity and its resemblances [12, 13], one may wonder if the model is self-consistent and the solution is reliable at such a high scale. Since the Hubble parameter during inflation is larger than M , the unitarity of the scattering amplitude seems to be violated during inflation. However, in the inflationary background, this estimation is not correct. Since the structures of the kinetic term and the interaction of the fluctuation around the background solution differ from the ones around the vacuum, the strong coupling scale depends on the background dynamics as well as the model parameter M , as also discussed in, *e.g.*, Ref. [14], in particular in other Higgs inflation models [1, 2, 15]. Fluctuations typically carry energies with the order of the Hubble scale in the quasi-de Sitter background, and hence the quantum fluctuations are under control if the Hubble parameter is smaller than the strong coupling scale.

In this letter, we investigate the strong coupling scale in the Higgs G-inflation. Here we identify the strong coupling scale as the scale where the tree-level unitarity is violated following the discussion in Ref. [13]. We show that in this criterion the system is sufficiently weakly coupled when the present horizon scales exited the horizon during inflation. Thus, the cosmological predictions evaluated in the previous studies [3, 4] are valid. However, we also find that the (tree-level) unitarity of the scattering amplitude is violated at a scale less than the Planck scale, which suggests that

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¹ It is also possible to generate a large gravitational wave background in the Higgs inflation with non-minimal coupling to gravity [9].

² Note that this term is *not* Galilean symmetric. We call it a “Galileon-like” term because it belongs to and is motivated by \mathcal{L}_3 in the generalized Galileon [6, 7] (or the Horndeski theory [8]).

the Higgs G-inflation cannot take place at the Planck scale, or at least we cannot describe how the Universe evolves when it starts from the Planck scale. Although the structure of the model is just a modification of chaotic inflation, the chaotic initial condition [16], where Universe starts from the Planck scale, $K \sim V \sim M_{\text{pl}}^4$, with K and V being the kinetic and potential energy, respectively, is problematic. Since the Higgs field value itself during inflation is larger than the strong coupling scale, it would be impossible to connect the results of the low-energy collider experiments to the precise values of running couplings at the inflationary scale without the knowledge of the UV physics behind the model, as is the case of the Higgs inflation with non-minimal coupling to gravity [13, 17, 18].

II. HIGGS G-INFLATION

First we summarize the Higgs G-inflation [4]. It is one of the Higgs inflation models where inflation is driven by the potential energy of the SM Higgs field \mathcal{H} with a Galileon-like derivative coupling, $(\mathcal{H}^\dagger D_\mu D^\mu \mathcal{H} + \text{h.c.})|D_\mu \mathcal{H}|^2/M^4$. The Lagrangian is given by

$$\mathcal{L} = \sqrt{-g} \left[\frac{M_{\text{pl}}^2}{2} R - |D_\mu \mathcal{H}|^2 - \left(\frac{\mathcal{H}^\dagger}{M^4} D_\mu D^\mu \mathcal{H} + \text{h.c.} \right) |D_\mu \mathcal{H}|^2 - \lambda |\mathcal{H}|^4 \right], \quad (1)$$

where R is the Ricci scalar, D_μ is the covariant derivative, and M and λ are model parameters whose mass dimensions are 1 and 0, respectively. Here we omit the Higgs mass term since it is irrelevant to the inflationary dynamics. We adopt the Friedman-Robertson-Walker metric, $ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$, with the Hubble parameter $H = \dot{a}/a$. Let us first investigate the physical Higgs in the unitary gauge, $\mathcal{H} = (0, \phi)/\sqrt{2}$. The Lagrangian for ϕ is given by

$$\mathcal{L} = \sqrt{-g} \left[\frac{M_{\text{pl}}^2}{2} R - \frac{1}{2} (D_\mu \phi)^2 - \frac{\phi D_\mu D^\mu \phi}{2M^4} (D_\mu \phi)^2 - \frac{\lambda}{4} \phi^4 \right]. \quad (2)$$

This system allows a potential-driven slow-roll inflation when the slow-roll conditions, $|\epsilon|, |\eta|, |\alpha| \ll 1$, are satisfied. Here the slow-roll parameters are given by

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \eta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}}, \quad \alpha \equiv \frac{\dot{\phi}}{H\phi}. \quad (3)$$

Note that we here introduced a new slow-roll parameter α to take into account the effect of the Galileon-like derivative coupling [4]. The slow-roll equations are found to be

$$3H^2 M_{\text{pl}}^2 = \frac{\lambda}{4} \phi^4, \quad (4)$$

$$3H\dot{\phi} \left(1 - \frac{3H\dot{\phi}\phi}{M^4} \right) + \lambda\phi^3 = 0. \quad (5)$$

Here we assumed that the effect of running of the model parameters is negligible and they are taken as constants. In particular, we focus on the case where $\lambda = \mathcal{O}(10^{-2}) > 0$ at the inflationary scale.³ For $\phi > \sqrt{2}\lambda^{-1/4}M$, the second term in the parenthesis in Eq. (5) dominates the first term, and we have the inflationary solution

$$\dot{\phi} = -\frac{2M^2 M_{\text{pl}}}{\sqrt{3}\phi}. \quad (6)$$

Inflation ends when the slow-roll condition $|\epsilon| \ll 1$ breaks,

$$\phi = \phi_{\text{end}} \equiv 2^{3/4} \lambda^{-1/8} \sqrt{MM_{\text{pl}}}. \quad (7)$$

The Higgs field value at the number of e -folds \mathcal{N} before the end of inflation is evaluated as

$$\phi_{\mathcal{N}} = (16\mathcal{N} + 8)^{1/4} \lambda^{-1/8} \sqrt{MM_{\text{pl}}}. \quad (8)$$

³ Though the recent experimental results [19] suggest the metastability of the Higgs potential [20], the parameter space where the Higgs quartic coupling is positive around the scale of the grand unified theory (GUT) is not excluded.

The power spectrum of the primordial scalar perturbation A_s is calculated as

$$A_s = \frac{(2\mathcal{N}+1)^2}{8\pi^2} \left(\frac{3}{8}\right)^{1/2} \lambda^{1/2} \left(\frac{M}{M_{\text{pl}}}\right)^2, \quad (9)$$

and its spectral tilt is evaluated as

$$n_s = 1 - \frac{4}{2\mathcal{N}+1}. \quad (10)$$

For the number of e -folds $\mathcal{N}_* \simeq 60$ where the pivot scale $k_0 = 0.05\text{Mpc}^{-1}$ exited the horizon during inflation, the measurement of the Planck satellite, $A_s \simeq 2.2 \times 10^{-9}$ [21], is reproduced by $M \simeq (\lambda/0.01)^{-1/4} \times 3 \times 10^{13}$ GeV. Consequently, during inflation the Higgs field value is around the GUT scale. The tensor-to-scalar ratio is given by⁴

$$r = \frac{64}{3} \left(\frac{2}{3}\right)^{1/2} \epsilon \simeq \frac{17}{2\mathcal{N}_*+1} \sim 0.14. \quad (11)$$

Since the model does not change the tensor sector compared to the standard scenario, the tensor spectral tilt is expressed by the slow-roll parameter as the standard expression $n_T \simeq -2\epsilon$. As a result, we have a non-standard consistency relation,

$$r \simeq -\frac{32\sqrt{6}}{9} n_T. \quad (12)$$

III. STRONG COUPLING IN THE INFLATON SELF-INTERACTION

One may wonder if this inflationary solution gives a consistent scenario. If the fluctuations around the inflationary trajectory couple too strongly, or the strong coupling scale (or the cutoff scale) is smaller than the Hubble scale, the above discussion is no longer reliable. In order to determine it, here we adopt the discussion in Ref. [13] (see also Ref. [14] for the similar discussion in the DGP model), where we identify the cutoff scale as the scale where the tree-level unitarity is violated [22]. Here we divide the Higgs field into the slowly varying classical part and excitations and estimate the cutoff scale by the power counting of the operators in the action expanded with respect to the canonically normalized excitations $\tilde{\chi}_i$.⁵ Once the operators are expanded as

$$\frac{\mathcal{O}_{(n)}(\chi_i, \partial\chi_i, \partial^2\chi_i)}{[\Lambda_{(n)}(\bar{\phi}, \dot{\phi})]^{n-4}}, \quad (13)$$

we expect that the tree-level unitarity is violated at $E_{\text{sc}} = \min\{\Lambda_{(n)}\}$. If it is sufficiently larger than the Hubble parameter during inflation, the semiclassical treatments of the model are valid and it is self-consistent.

First we see the self-interaction of the fluctuation along the inflationary trajectory. Let us rewrite the inflaton field as

$$\phi(x) \rightarrow \bar{\phi}(t) + \chi(x), \quad (14)$$

where $\bar{\phi}(t)$ is the homogeneous background solution and $\chi(x)$ is the fluctuation around the background. Noting that

$$D_\mu D^\mu \phi = -\ddot{\phi} - 3H\dot{\phi} + \square\chi, \quad (D_\mu \phi)^2 = -\dot{\phi}^2 - 2\dot{\phi}\dot{\chi} + (D_\mu \chi)^2, \quad (15)$$

⁴ This value of the tensor-to-scalar ratio is disfavored by the recent results of Planck. However, here we neglect the effect of the running coupling constant, which may be able to reduce it slightly to reconcile with the observation.

⁵ The subscript i is introduced to take into account the Nambu-Goldstone (NG) modes.

we can expand the derivative coupling as

$$\begin{aligned}
& -\frac{1}{2}(D_\mu\phi)^2 - \frac{1}{2M^4}\phi\Box\phi(D_\mu\phi)^2 \\
& = \frac{1}{2}\left(1 - \frac{\bar{\phi}(\ddot{\phi} + 3H\dot{\phi})}{M^4}\right)\dot{\phi}^2 - \frac{(\ddot{\phi} + 3H\dot{\phi})\dot{\phi}^2}{2M^4}\chi + \left(1 - \frac{\bar{\phi}(\ddot{\phi} + 3H\dot{\phi})}{M^4}\right)\dot{\phi}\dot{\chi} + \frac{\bar{\phi}\dot{\phi}^2}{2M^4}\Box\chi \\
& + \frac{\dot{\phi}^2}{2M^4}\chi\Box\chi - \frac{1}{2}\left(1 - \frac{\bar{\phi}(\ddot{\phi} + 3H\dot{\phi})}{M^4}\right)(D_\mu\chi)^2 - \frac{\dot{\phi}(\ddot{\phi} + 3H\dot{\phi})}{M^4}\chi\dot{\chi} + \frac{\bar{\phi}\dot{\phi}}{M^4}\dot{\chi}\Box\chi \\
& + \frac{\dot{\phi}}{M^4}\chi\dot{\chi}\Box\chi + \frac{\ddot{\phi} + 3H\dot{\phi}}{2M^4}\chi(D_\mu\chi)^2 - \frac{\bar{\phi}}{2M^4}\Box\chi(D_\mu\chi)^2 - \frac{1}{2M^4}\chi\Box\chi(D_\mu\chi)^2.
\end{aligned} \tag{16}$$

With taking a partial integral, the kinetic term of χ is given by

$$\begin{aligned}
S_{\text{kin}}[\chi] & = \int d^4x a^3(t) \left(\frac{\dot{\phi}^2}{2M^4}\chi\Box\chi - \frac{1}{2}\left(1 - \frac{\bar{\phi}(\ddot{\phi} + 3H\dot{\phi})}{M^4}\right)(D_\mu\chi)^2 + \frac{\bar{\phi}\dot{\phi}}{M^4}\dot{\chi}\Box\chi \right) \\
& = \int d^4x a^3(t) \left[\frac{1}{2}\left(1 + \frac{2\dot{\phi}^2 - 6H\bar{\phi}\dot{\phi}}{M^4}\right)\dot{\chi}^2 - \frac{1}{2}\left(1 - \frac{2\bar{\phi}(\ddot{\phi} + 2H\dot{\phi})}{M^4}\right)\frac{\delta^{ij}}{a^2(t)}\partial_i\chi\partial_j\chi \right] \\
& \quad + (\text{mass term}) + (\text{total derivative}).
\end{aligned} \tag{17}$$

The inflaton fluctuation can be canonically normalized as follows. Defining

$$G(t) \equiv 1 + \frac{2\dot{\phi}^2 - 6H\bar{\phi}\dot{\phi}}{M^4}, \quad F(t) \equiv 1 - \frac{2\bar{\phi}(\ddot{\phi} + 2H\dot{\phi})}{M^4} \tag{18}$$

and

$$d\tilde{x}^i \equiv \sqrt{\frac{G(t)}{F(t)}}a(t)dx^i, \quad \tilde{\chi} \equiv \frac{F(t)^{3/4}}{G(t)^{1/4}}\chi, \tag{19}$$

we see that $\tilde{\chi}$ is canonically normalized,

$$S_{\text{kin}}[\chi] = \int dt d^3\tilde{x} \frac{1}{2} \left(\dot{\tilde{\chi}}^2 - \delta^{ij} \frac{\partial\tilde{\chi}}{\partial\tilde{x}^i} \frac{\partial\tilde{\chi}}{\partial\tilde{x}^j} \right) + (\text{mass term}) + (\text{total derivative}). \tag{20}$$

Note that there arise terms coming from the derivatives of $F(t)$ and $G(t)$, but they are mass terms and total derivatives, and hence we do not write explicitly. With the above coordinate and field redefinition, higher derivative interactions are rewritten as

$$\begin{aligned}
S_{\text{int}} & = \int dt d^3\tilde{x} \frac{1}{G(t)^{3/4}F(t)^{3/4}} \frac{\dot{\phi}}{M^4} \tilde{\chi} \dot{\tilde{\chi}} \Box\tilde{\chi} + \frac{1}{G(t)^{3/4}F(t)^{3/4}} \frac{\ddot{\phi} + 3H\dot{\phi}}{2M^4} \tilde{\chi} (D_\mu\tilde{\chi})^2 \\
& \quad - \frac{1}{G(t)^{3/4}F(t)^{3/4}} \frac{\bar{\phi}}{2M^4} \Box\tilde{\chi} (D_\mu\tilde{\chi})^2 - \frac{1}{G(t)^{1/2}F(t)^{3/2}} \frac{1}{2M^4} \chi \Box\chi (D_\mu\chi)^2.
\end{aligned} \tag{21}$$

Here we again omit the terms coming from the derivatives of $F(t)$ and $G(t)$. But these terms are slow-roll suppressed, and hence they are less dangerous than the terms in Eq. (21) in the inflationary background. We also do not write the modification of spatial derivative explicitly, which changes the structure of $\Box\tilde{\chi}$ and $(D_\mu\tilde{\chi})^2$ slightly since $F(t) \sim G(t)$, and hence the phase velocity is of the order of the unity if there are not any non-trivial cancellations, (which is true for the slow-roll inflationary solution as we will see below). Consequently, we identify the strong coupling scale for the $\tilde{\chi}$ field as

$$E > E_{\text{sc}} \equiv \min. \left\{ \frac{G(t)^{3/8}F(t)^{3/8}M^2}{\dot{\phi}^{1/2}}, \frac{2G(t)^{3/4}F(t)^{3/4}M^4}{\ddot{\phi} + 3H\dot{\phi}}, \frac{2^{1/3}G(t)^{1/4}F(t)^{1/4}M^{4/3}}{\bar{\phi}^{1/3}}, 2^{1/4}G(t)^{1/8}F(t)^{3/8}M \right\}. \tag{22}$$

During inflation, we have the slow-roll trajectory,

$$H \simeq \frac{\lambda^{1/2} \bar{\phi}^2}{2\sqrt{3}M_{\text{pl}}}, \quad \dot{\bar{\phi}} = -\frac{2M^2 M_{\text{pl}}}{\sqrt{3}\bar{\phi}}, \quad \ddot{\bar{\phi}} = -\frac{4M^4 M_{\text{pl}}^2}{3\bar{\phi}^3}, \quad |H\dot{\bar{\phi}}| \gg |\ddot{\bar{\phi}}|, \quad (23)$$

and hence

$$G(t) \simeq -\frac{6H\bar{\phi}\dot{\bar{\phi}}}{M^4} \simeq \frac{2\lambda^{1/2}\bar{\phi}^2}{M^2}, \quad F(t) \simeq -\frac{4H\bar{\phi}\dot{\bar{\phi}}}{M^4} \simeq \frac{4\lambda^{1/2}\bar{\phi}^2}{3M^2}. \quad (24)$$

As a result, we find that the strong coupling scale is given by

$$E_{\text{sc}}(\bar{\phi}) = \frac{2^{1/3}G(t)^{1/4}F(t)^{1/4}M^{4/3}}{\bar{\phi}^{1/3}} \simeq \lambda^{1/4}\bar{\phi}^{2/3}M^{1/3} \quad (25)$$

for $\bar{\phi} > \phi_{\text{end}} \sim \sqrt{MM_{\text{pl}}}$. Therefore, for

$$H(\bar{\phi}) \gg E_{\text{sc}}(\bar{\phi}) \Leftrightarrow \bar{\phi} \gg \lambda^{-3/16}M^{1/4}M_{\text{pl}}^{3/4} \simeq 0.1 \left(\frac{\lambda}{0.01}\right)^{-3/16} \left(\frac{M}{10^{13}\text{GeV}}\right)^{1/4} M_{\text{pl}} \equiv \bar{\phi}_{\text{sc}}, \quad (26)$$

the system is strongly coupled compared to the inflationary scale. Since $\phi_{\mathcal{N}_*} \ll \bar{\phi}_{\text{sc}}$ for $\mathcal{N}_* \simeq 60$, the χ field is weakly self-interacted when the scale we now see in the CMB exited the horizon during inflation. This constraint also suggests that the Higgs G-inflation cannot start from the Higgs field value larger than the Planck scale, unlike the usual chaotic inflation. Note that for

$$H^2(\bar{\phi}) \ll \frac{F(\bar{\phi})^{3/4}}{G(\bar{\phi})^{1/4}}\dot{\bar{\phi}} \Leftrightarrow \bar{\phi} \ll \lambda^{-3/16}M^{1/4}M_{\text{pl}}^{3/4} \simeq \bar{\phi}_{\text{sc}}, \quad (27)$$

quantum fluctuations are small compared with the classical evolution of the inflaton. Therefore the Higgs G-inflation cannot have the stage of eternal inflation.

Note that here we neglect the mixing between the inflaton fluctuation and scalar perturbation of the metric tensor. However, they are decoupled in the $M_{\text{pl}} \rightarrow \infty$ limit with $\lambda\phi^4/M_{\text{pl}}^2 = \text{const.}$ We can see that the quadratic action of the scalar metric perturbation in the unitary gauge [4] differs from the action of inflaton fluctuations only by Planck suppressed terms, which are subdominant during inflation. Therefore, the mixing between the inflaton fluctuation and the scalar perturbation of the metric tensor is negligible for our purposes.

IV. STRONG COUPLING IN THE NAMBU-GOLDSTONE SECTOR

Since the transverse modes of gauge bosons and gravitons do not change their kinetic term by the Galileon-like Higgs derivative coupling, the strong coupling scale in and between these sectors is equal to or larger than the one in the Higgs self-interaction. However, the kinetic terms of the longitudinal mode of the gauge bosons or the NG modes are modified by the Galileon-like derivative coupling. Therefore we cannot tell if the system is weakly coupled during inflation unless we also check the interaction of NG modes. Here we investigate the interaction of the NG modes along the inflationary trajectory.

Let us expand the Higgs field along the inflationary trajectory as

$$\mathcal{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} \theta_2 + i\theta_3 \\ \bar{\phi}(t) + \chi(x) + i\theta_1(x) \end{pmatrix}. \quad (28)$$

Here χ is the inflaton fluctuation and θ_i are the NG modes. By taking a partial integral, the action for the inflaton

fluctuations χ and the NG modes θ_i is written by

$$\begin{aligned}
S_{\text{kin}} = \int d^4x a^3(t) & \left[\frac{1}{2} \left(1 + \frac{2\dot{\bar{\phi}}^2 - 6H\bar{\phi}\dot{\bar{\phi}}}{M^4} \right) \dot{\chi}^2 - \frac{1}{2} \left(1 - \frac{2\bar{\phi}(\ddot{\bar{\phi}} + 2H\dot{\bar{\phi}})}{M^4} \right) \frac{\delta^{ij}}{a^2(t)} \partial_i \chi \partial_j \chi \right. \\
& - \frac{1}{2} \left(1 - \frac{\bar{\phi}\ddot{\bar{\phi}} - \dot{\bar{\phi}}^2 + 3H\bar{\phi}\dot{\bar{\phi}}}{M^4} \right) \left(\sum_i (\partial_\mu \theta_i)^2 \right) \\
& + \frac{1}{M^4} (\ddot{\bar{\phi}}\chi - \bar{\phi}\partial_\mu \partial^\mu \chi) \left((\partial_\mu \chi)^2 + \sum_i (\partial_\mu \theta_i)^2 \right) + \frac{\dot{\bar{\phi}}}{M^4} \dot{\chi} \left(\chi \partial_\mu \partial^\mu \chi + \sum_i \theta_i \partial_\mu \partial^\mu \theta_i \right) \\
& \left. - \frac{1}{2M^4} \left((\partial_\mu \chi)^2 + \sum_i (\partial_\mu \theta_i)^2 \right) \left(\chi \partial_\mu \partial^\mu \chi + \sum_i \theta_i \partial_\mu \partial^\mu \theta_i \right) \right] + (\text{potential terms}) + (\text{total derivative}).
\end{aligned} \tag{29}$$

During inflationary stage, we have $3H\bar{\phi}\dot{\bar{\phi}} \gg, \dot{\bar{\phi}}^2, \bar{\phi}\ddot{\bar{\phi}}, M^4$, and hence the action is approximated as

$$\begin{aligned}
S_{\text{kin}} \simeq \int d^4x a^3(t) & \left[-\frac{1}{2} \frac{6H\bar{\phi}\dot{\bar{\phi}}}{M^4} \left(\dot{\chi}^2 - \frac{2}{3} \frac{\delta^{ij}}{a^2(t)} \partial_i \chi \partial_j \chi \right) + \frac{1}{2} \frac{3H\bar{\phi}\dot{\bar{\phi}}}{M^4} \left(\sum_i \left(\dot{\theta}_i^2 - \frac{\delta^{jk}}{a^2(t)} \partial_j \theta_i \partial_k \theta_i \right) \right) \right. \\
& + \frac{1}{M^4} (\ddot{\bar{\phi}}\chi - \bar{\phi}\partial_\mu \partial^\mu \chi) \left((\partial_\mu \chi)^2 + \sum_i (\partial_\mu \theta_i)^2 \right) + \frac{\dot{\bar{\phi}}}{M^4} \dot{\chi} \left(\chi \partial_\mu \partial^\mu \chi + \sum_i \theta_i \partial_\mu \partial^\mu \theta_i \right) \\
& \left. - \frac{1}{2M^4} \left((\partial_\mu \chi)^2 + \sum_i (\partial_\mu \theta_i)^2 \right) \left(\chi \partial_\mu \partial^\mu \chi + \sum_i \theta_i \partial_\mu \partial^\mu \theta_i \right) \right] + (\text{potential terms}) + (\text{total derivative}).
\end{aligned} \tag{30}$$

We can see that the NG modes have almost the same kinetic term structures to that of the inflaton fluctuation. Consequently, NG modes are canonically normalized with subluminal sound speeds by almost the same field redefinition to the inflaton fluctuations. As a result, the strong coupling scale in the NG boson sector is also the same to the χ self-interaction, *i.e.*,

$$E_{\text{sc}}(\bar{\phi}) \simeq \lambda^{1/4} \bar{\phi}^{2/3} M^{1/3}. \tag{31}$$

We here conclude that the field dependent cutoff scale of the Higgs G-inflation model estimated by the power counting of all the fluctuation operators is sufficiently larger than the energy scale carried by the fluctuations. Therefore, the requirement for the validity of the semiclassical treatment performed in the previous studies [3, 4] is fulfilled. Note that the derivative interactions between the Higgs and the NG modes induce ghost-like degrees of freedom [23], but it is harmless since the scale at which they appear is also the strong coupling scale, which is larger than the Hubble parameter during inflation.

V. SUMMARY AND DISCUSSION

In this letter, we evaluate the strong coupling scale in the Higgs G-inflation identifying it as the scale where the tree-level unitarity is violated. We find that the strong coupling scale of the fluctuations around the inflationary trajectory depends on the background dynamics and is larger than the Hubble scale when the present horizon scale exited the horizon. Since the inflationary background modifies the structure of the kinetic term of the fluctuations, the mass parameter in the original Lagrangian is not directly related to the strong coupling scale. As a result, the semiclassical calculation performed in the previous studies [3, 4] are valid and the model is self-consistent. Note that the strong coupling scale meets the Hubble scale at $\bar{\phi} \simeq \lambda^{-3/16} M^{1/4} M_{\text{pl}}^{3/4} (< M_{\text{pl}})$. This suggests that we cannot describe the onset of the Higgs G-inflation from the chaotic initial condition, where both potential and kinetic energies are of the order of the Planck scale. For the related study on the generalized G-inflation [7], see Ref. [11].

Note that the condition that we adopt in this letter is the necessary condition, but not the sufficient condition [13]. Since the Galileon-like derivative coupling is a non-renormalizable operator, quantum corrections generate an infinite number of higher order interactions. To remove the divergences, we need to add an infinite number of counter terms.

In order to tell the sufficient condition of the validity of the model, we need to examine how these loop corrections can be suppressed, but it is a matter of “naturalness”. In the case of the Higgs inflation with non-minimal coupling to gravity, the theory has an asymptotic scale invariance, which guarantees the absences of higher order terms, and hence it is “natural” [13]. The naturalness in $P(X)$ and Galileon theories that have additional symmetries like Galilean symmetry or shift symmetry is studied in Ref. [24]. However, the higher derivative term we introduce in the Higgs G-inflation does not have known (asymptotic) symmetry behind it. Note that the “Galileon-like” term as well as the potential term is not Galilean symmetric. Nevertheless, the fact that the strong coupling scale is large enough for the Higgs G-inflation may suggest that the existence of a hidden asymptotic symmetry, which makes the model “natural”. But further investigations are needed which is left for the future study.

Apart from the naturalness, what we find here has an important insight in the connection between the inflationary parameters and low-energy physics. Since the Higgs field value during inflation is larger than the strong coupling scale, there can appear higher order interactions to modify the running of the couplings or at least threshold effect between the electroweak scale and the inflationary scale, as are discussed in Refs. [13, 17, 18]. Therefore, their connection is sensitive to the detail of the UV completion of the theory. In particular, even if the low-energy experiments will suggest the meta-stability of the Higgs potential [20, 25], the Higgs G-inflation may be still possible depending on the UV completion as is the case discussed recently in Ref. [18]. Note that there are discussions on the UV completion of the (generalized) Galileons. Since the (generalized) Galileon theories exhibit low-energy superluminalities of fluctuations in specific backgrounds, it may suggest the absence of any local Lorentz invariant UV completions [26]. However, it is still a subtle issue [27–29], and hence careful studies on the UV completion are needed for the detailed investigation of the Higgs G-inflation.

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